



# The all-electron full-potential linearized augmented plane-wave method

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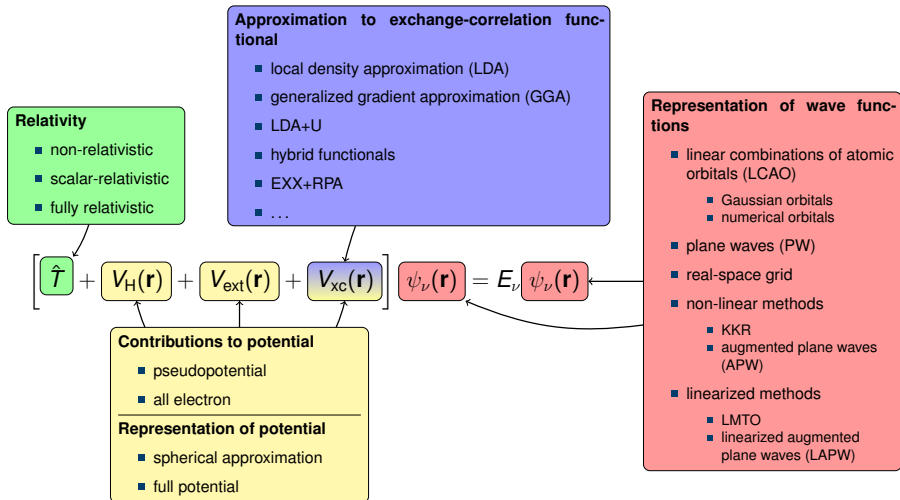
# Outline

- **Theoretical background**
  - The FLAPW method and the LAPW basis
  - Separation of core electrons from valence electrons
  - Representation of density and potential
- **FLAPW in practice**
  - Setting the parameters
  - Semicore states and ghost bands
  - The linearization error
- **Using fleur**
  - The input file generator
  - The inp.xml file
- **Further reading**
- **Conclusion**

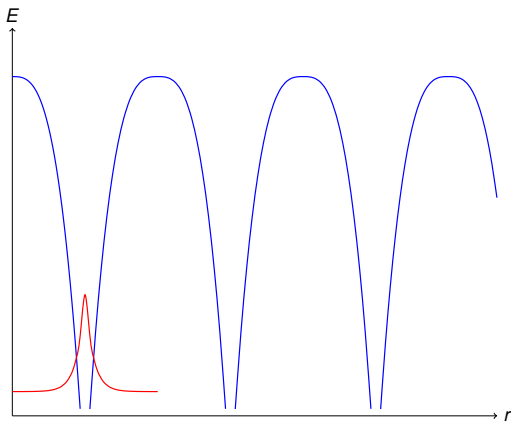
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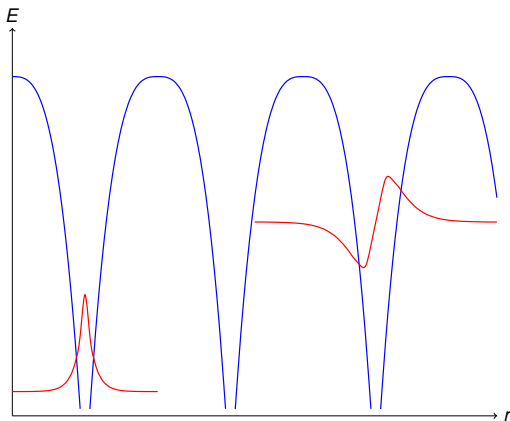
# Motivation: FLAPW in zoo of electronic structure methods



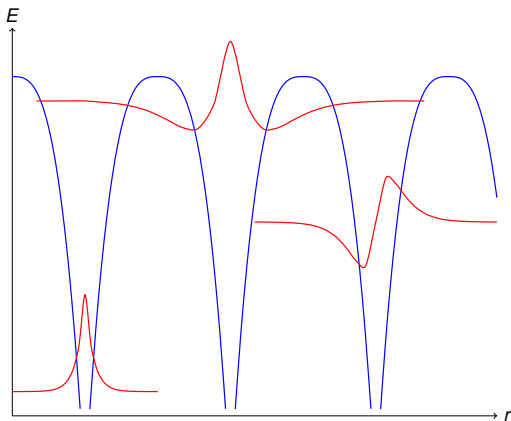
# Potential and wave functions in a crystal



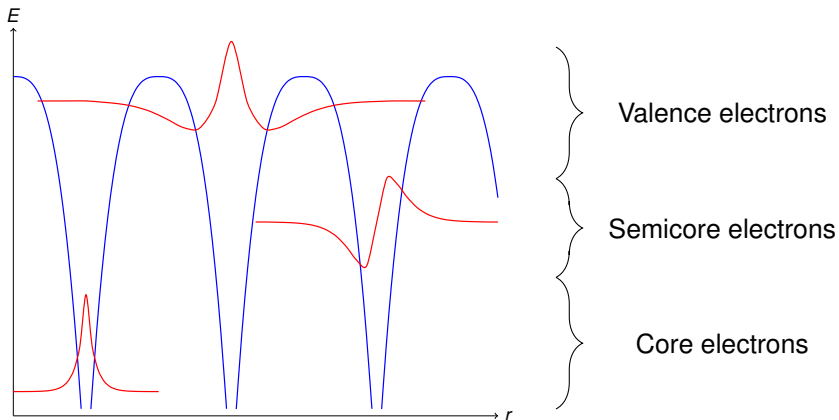
# Potential and wave functions in a crystal



# Potential and wave functions in a crystal

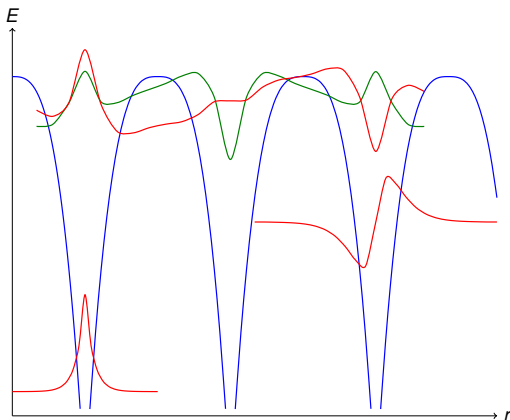


# Potential and wave functions in a crystal





# Potential and wave functions in a crystal



Bloch theorem:

If  $V(\mathbf{r}) = V(\mathbf{r} + \mathbf{R})$ :

$$\Psi(\mathbf{r}) = e^{i\mathbf{k}\mathbf{r}} \cdot u_{\mathbf{k}}(\mathbf{r}),$$

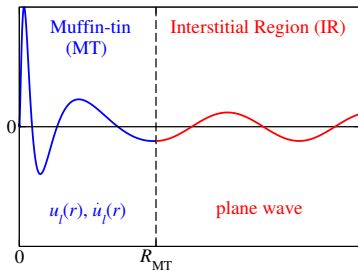
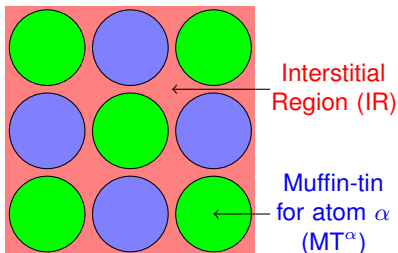
$$u_{\mathbf{k}}(\mathbf{r}) = u_{\mathbf{k}}(\mathbf{r} + \mathbf{R})$$

# The LAPW basis

- Atom-centered functions in MT spheres matched in value and slope to plane waves in interstitial region (IR)

$$\phi_{\mathbf{k}\mathbf{G}}(\mathbf{r}) = \begin{cases} \frac{1}{\sqrt{\Omega}} e^{i(\mathbf{k}+\mathbf{G})\mathbf{r}} & \text{for } \mathbf{r} \in \text{IR} \\ \sum_L [a_{\mathbf{k}\mathbf{G}}^{L\alpha} u_l^\alpha(r_\alpha, E_l^\alpha) + b_{\mathbf{k}\mathbf{G}}^{L\alpha} \dot{u}_l^\alpha(r_\alpha, E_l^\alpha)] Y_L(\hat{\mathbf{r}}_\alpha) & \text{for } \mathbf{r} \in \text{MT}^\alpha \end{cases}$$

- $u_l^\alpha$  and  $\dot{u}_l^\alpha$  are solutions and energy derivatives for the spherical potential at energy parameters  $E_l^\alpha$



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- $u_l^\alpha$  and  $\dot{u}_l^\alpha$  are solutions and energy derivatives for the spherical potential at energy parameters  $E_l^\alpha$

Parameters:

- $K_{\max} = |\mathbf{k} + \mathbf{G}|_{\max}$  reciprocal plane wave cutoff
- $l_{\max}^\alpha$  angular momentum cutoff for sphere  $\alpha$
- $R_{\text{MT}}^\alpha$  radius for muffin-tin sphere  $\alpha$
- $E_l^\alpha$  energy parameter for  $u_l^\alpha, \dot{u}_l^\alpha$

Note: LAPW basis depends on atom positions and features discontinuities at MT boundaries

# Orthogonality of LAPW basis functions to core electron states (1)

- $u_l(r)$ ,  $u_l^c(r)$  given by radial Schrödinger equation:

$$\left[ -\frac{1}{2} \frac{\partial^2}{\partial r^2} + \frac{l(l+1)}{2r^2} + V_{\text{eff}}^{\text{sphr}}(r) \right] ru_l(r) = E_l ru_l(r) \quad (1)$$

- Multiply (1) for  $u_l(r)$  by  $ru_l^c(r)$  and vice versa, subtract the two resulting equations from each other, and integrate:

$$\int_0^{R_{\text{MT}}} -\frac{1}{2} ru_l^c(r) \frac{\partial^2}{\partial r^2} ru_l(r) + \frac{1}{2} ru_l(r) \frac{\partial^2}{\partial r^2} ru_l^c(r) dr = (E_l - E_l^c) \int_0^{R_{\text{MT}}} u_l(r) r^2 u_l^c(r) dr$$

- Assumption:  $u_l^c(r)|_{R_{\text{MT}}} = 0$ ,  $\frac{\partial}{\partial r} u_l^c(r)|_{R_{\text{MT}}} = 0$
- We obtain:  $0 = \langle u_l^c | u_l \rangle_{R_{\text{MT}}}$  and analogously  $0 = \langle u_l^c | \dot{u}_l \rangle_{R_{\text{MT}}}$

# Orthogonality of LAPW basis functions to core electron states (2)

- Orthogonality allows to determine core and valence electron energies and wave functions separately from each other
- Core electrons
  - Representation for each atom separately on radial mesh
  - Fully relativistic treatment
- Valence electrons
  - Representation by LAPW basis
  - Scalar-relativistic description in MT spheres
  - Optional inclusion of spin-orbit coupling
- But: assumption  $u_l^c(r)|_{R_{\text{MT}}} = 0, \frac{\partial}{\partial r} u_l^c(r)|_{R_{\text{MT}}} = 0$  only approximately fulfilled
  - Semicore states can lead to ghost bands

# The linearization within the LAPW basis

- Description in MT spheres is not systematically improved by increasing the reciprocal cutoff parameter  $K_{\max}$

## Linearization of solutions $u_l$ at arbitrary energy $\epsilon$

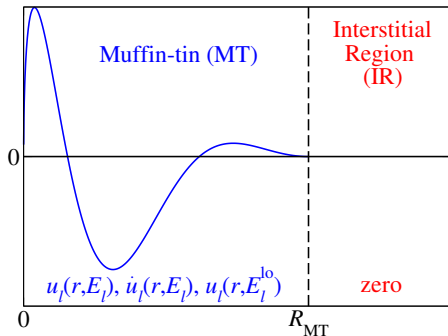
- $u_l^\alpha(r_\alpha, \epsilon) = u_l^\alpha(r_\alpha, E_l^\alpha) + (\epsilon - E_l^\alpha)\dot{u}_l^\alpha(r_\alpha, E_l^\alpha) + \mathcal{O}[(\epsilon - E_l^\alpha)^2]$
- Due to the restriction to the function space spanned by  $u_l^\alpha(r_\alpha, E_l^\alpha)$  and  $\dot{u}_l^\alpha(r_\alpha, E_l^\alpha)$  we obtain a linearization error.
- This description is sufficient to obtain accurate results for many materials.

# Extending the LAPW basis with local orbitals

## Additional basis functions localized in MT spheres

$$\phi_L^{\text{lo}}(\mathbf{r}) = [a_L^{\text{lo}} u_l^\alpha(r_\alpha, E_l^\alpha) + b_L^{\text{lo}} \dot{u}_l^\alpha(r_\alpha, E_l^\alpha) + c_L^{\text{lo}} u_l^\alpha(r_\alpha, E_l^{\text{lo}})] Y_L(\hat{\mathbf{r}}_\alpha)$$

- Mainly used to describe semicore states
- Determination of  $a_L^{\text{lo}}$ ,  $b_L^{\text{lo}}$ , and  $c_L^{\text{lo}}$  by enforcing zero value and slope at the MT boundary, as well as a normalization condition on the local orbital

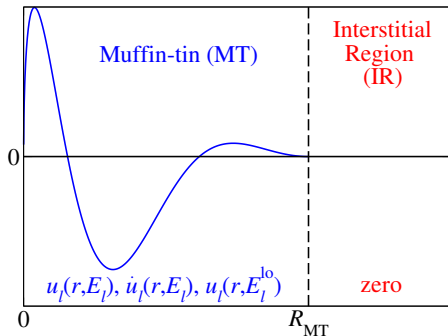


# Extending the LAPW basis with local orbitals

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$$\phi_L^{\text{lo}}(\mathbf{r}) = [a_L^{\text{lo}} u_l^\alpha(r_\alpha, E_l^\alpha) + b_L^{\text{lo}} \dot{u}_l^\alpha(r_\alpha, E_l^\alpha) + c_L^{\text{lo}} u_l^\alpha(r_\alpha, E_l^{\text{lo}})] Y_L(\hat{\mathbf{r}}_\alpha)$$

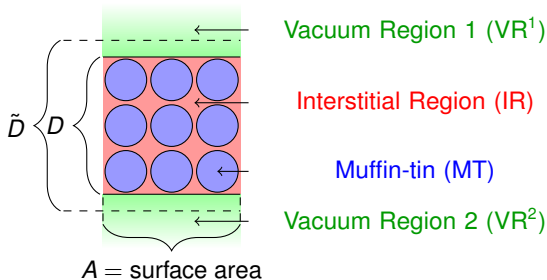
- Semicore states (SCLO)
  - Choose  $E_l^{\text{lo}}$  to be energy of semicore state
- Unoccupied orbitals (HELO)
  - Choose  $E_l^{\text{lo}}$  above Fermi energy
- Higher derivative LOs (HDLO)
  - Choose  $\dot{u}_l^\alpha(r_\alpha, E_l^\alpha)$  instead of  $u_l^\alpha(r_\alpha, E_l^{\text{lo}})$





# The LAPW basis for films

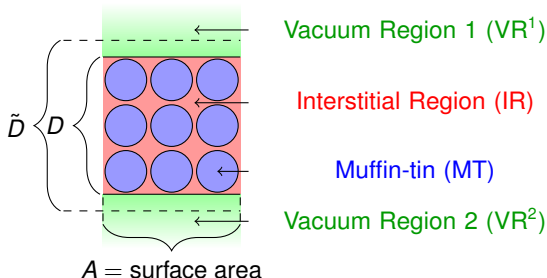
$$\phi_{\mathbf{k}_{\parallel}\mathbf{G}}(\mathbf{r}) = \begin{cases} \frac{1}{\sqrt{\Omega}} e^{i(\mathbf{k}_{\parallel} + \mathbf{G})\mathbf{r}} & \text{for } \mathbf{r} \in \text{IR} \\ \sum_L \left[ a_{\mathbf{k}_{\parallel}\mathbf{G}}^{L\alpha} u_l^\alpha(r_\alpha, E_l^\alpha) + b_{\mathbf{k}_{\parallel}\mathbf{G}}^{L\alpha} \dot{u}_l^\alpha(r_\alpha, E_l^\alpha) \right] Y_L(\hat{\mathbf{r}}_\alpha) & \text{for } \mathbf{r} \in \text{MT}^\alpha \\ \left[ a_{\mathbf{k}_{\parallel}\mathbf{G}}^{\text{vac}} u_{\mathbf{k}_{\parallel}\mathbf{G}_{\parallel}}^{\text{vac}}(z, E^{\text{vac}}) + b_{\mathbf{k}_{\parallel}\mathbf{G}}^{\text{vac}} \dot{u}_{\mathbf{k}_{\parallel}\mathbf{G}_{\parallel}}^{\text{vac}}(z, E^{\text{vac}}) \right] \\ \times \frac{1}{\sqrt{A}} e^{i(\mathbf{k}_{\parallel} + \mathbf{G}_{\parallel})\mathbf{r}_{\parallel}} & \text{for } \mathbf{r} \in \text{VR}^{\text{vac}} \end{cases}$$



- $u_{\mathbf{k}_{\parallel}\mathbf{G}_{\parallel}}^{\text{vac}}, \dot{u}_{\mathbf{k}_{\parallel}\mathbf{G}_{\parallel}}^{\text{vac}}$ : solutions, energy derivatives to vacuum potential at energy parameters  $E^{\text{vac}}$
- $G_{\perp} = 2\pi n / \tilde{D}$

# The LAPW basis for films

$$\phi_{\mathbf{k}_{\parallel} \mathbf{G}}(\mathbf{r}) = \begin{cases} \frac{1}{\sqrt{\Omega}} e^{i(\mathbf{k}_{\parallel} + \mathbf{G})\mathbf{r}} & \text{for } \mathbf{r} \in \text{IR} \\ \sum_L \left[ a_{\mathbf{k}_{\parallel} \mathbf{G}}^{L\alpha} u_l^\alpha(r_\alpha, E_l^\alpha) + b_{\mathbf{k}_{\parallel} \mathbf{G}}^{L\alpha} \dot{u}_l^\alpha(r_\alpha, E_l^\alpha) \right] Y_L(\hat{\mathbf{r}}_\alpha) & \text{for } \mathbf{r} \in \text{MT}^\alpha \\ \left[ a_{\mathbf{k}_{\parallel} \mathbf{G}}^{\text{vac}} u_{\mathbf{k}_{\parallel} \mathbf{G}_{\parallel}}^{\text{vac}}(z, E^{\text{vac}}) + b_{\mathbf{k}_{\parallel} \mathbf{G}}^{\text{vac}} \dot{u}_{\mathbf{k}_{\parallel} \mathbf{G}_{\parallel}}^{\text{vac}}(z, E^{\text{vac}}) \right] \\ \times \frac{1}{\sqrt{A}} e^{i(\mathbf{k}_{\parallel} + \mathbf{G}_{\parallel})\mathbf{r}_{\parallel}} & \text{for } \mathbf{r} \in \text{VR}^{\text{vac}} \end{cases}$$



Parameters:

- $D$  - vacuum boundary
- $\tilde{D}$  - determination of  $G_{\perp}$
- $E^{\text{vac}}$  - vacuum energy parameters

# Representation of density and potential

## Plane-wave part

$$\rho^{\text{PW}}(\mathbf{r}) = \sum_{\mathbf{G}}^{\mathbf{G}_{\text{max}}} \rho_{\mathbf{G}} \cdot e^{i\mathbf{G}\mathbf{r}}$$

- Actually represented by stars
  - Linear combinations of plane waves according to symmetry

## MT sphere $\alpha$

$$\rho^{\alpha}(\mathbf{r}) = \sum_L^{L_{\text{max}}^{\alpha}} \rho_L^{\alpha}(r_{\alpha}) Y_L(\hat{\mathbf{r}}_{\alpha})$$

- Actually represented by lattice harmonics
  - Linear combinations of spherical harmonics according to symmetry

Parameters:

- $\mathbf{G}_{\text{max}}$  reciprocal plane-wave cutoff for density, potential
- $\mathbf{G}_{\text{maxXC}}$  reduced cutoff for exchange correlation potential

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# Choice of $K_{\max}$ , $l_{\max}^{\alpha}$ , and $l_{\text{nonsphr}}^{\alpha}$

- Rayleigh expansion of planes waves at MT boundary:

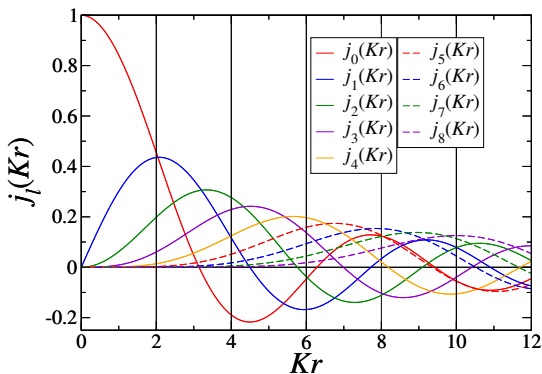
$$e^{i\mathbf{K}r} = 4\pi e^{i\mathbf{K}\tau_{\alpha}} \sum_L i^l Y_L^*(\hat{\mathbf{K}}) j_l(Kr_{\alpha}) Y_L(\hat{\mathbf{r}}_{\alpha}) \Big|_{r_{\alpha}=R_{\text{MT}}^{\alpha}}$$

- Rule of thumb:

$$l_{\max}^{\alpha} \approx K_{\max} \cdot R_{\text{MT}}^{\alpha} \\ \approx 8 - 12$$

- Determination of  $K_{\max}$  based on  $R_{\text{MT}}^{\alpha, \min}$

- $l_{\text{nonsphr}}^{\alpha} \approx \min(8, l_{\max}^{\alpha} - 2)$



# Choice of $G_{\max}$ and $G_{\max XC}$

- $G_{\max}$  is cutoff for different functions
  - Plane wave part of charge density  $\rho^{\text{PW}}(\mathbf{r})$
  - Plane wave part of potential  $V_{\text{eff}}^{\text{PW}}(\mathbf{r})$
  - Step function  $\Theta(\mathbf{r})$  indicating the interstitial region
- $V_{\text{eff}}^{\text{PW}}(\mathbf{r})$  and  $\Theta(\mathbf{r})$  have infinitely many plane-wave coefficients.
- Interstitial potential contribution to Hamilton matrix:

$$\langle \phi_{\mathbf{kG}} | \Theta(\mathbf{r}) V_{\text{eff}}^{\text{PW}}(\mathbf{r}) | \phi_{\mathbf{kG}'} \rangle$$

- Rule

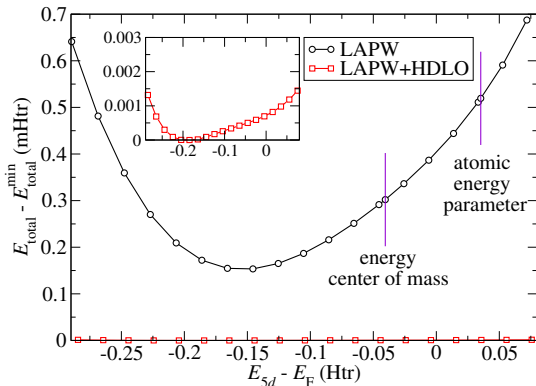
$$G_{\max} \geq G_{\max XC} \geq 2 \cdot K_{\max}$$

- typically  $G_{\max} \approx 3 \cdot K_{\max}$ ,  $G_{\max XC} \approx 2.5 \cdot K_{\max}$

# Choice of the energy parameters

- energy center of mass of the  $l$ -projected DOS
  - minimizes quadratic error weighted by charge in each eigenstate
- atomic solutions
  - yields more friendly convergence behavior

fcc Ce



# Choice of MT radii

- Due to different bonding lengths in different materials the  $R_{\text{MT}}$  are material dependent.
- If calculations have to be compared identical MT radii should be chosen.

## Large MT radii

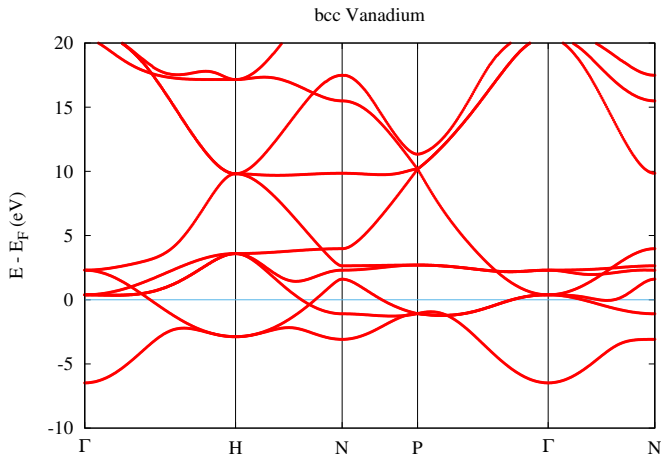
- Faster calculations
- Larger linearization error
- Fewer SCLOs needed
- Some quantities only evaluated in MT

## Small MT radii

- Slower calculations
- More stable calculations
- Smaller linearization error
- More SCLOs needed
- More space available for structural relaxations

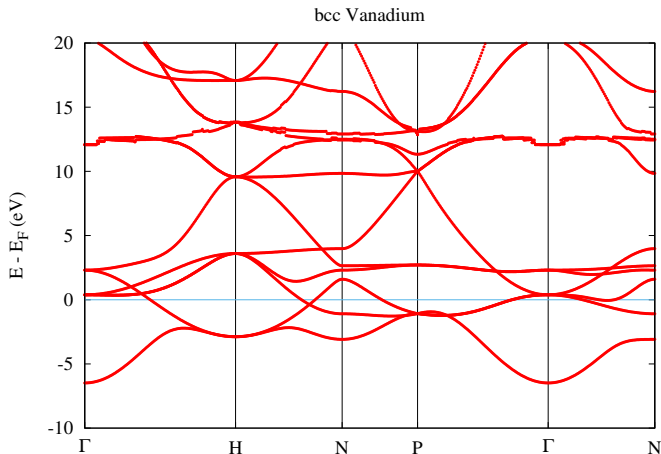


# Semicore states and ghost bands



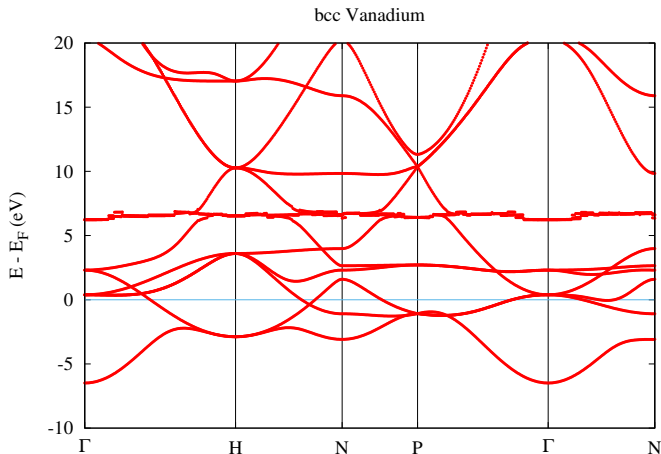
- $R_{MT} = 2.25 a_0$ , lostElectrons = 0.086

# Semicore states and ghost bands



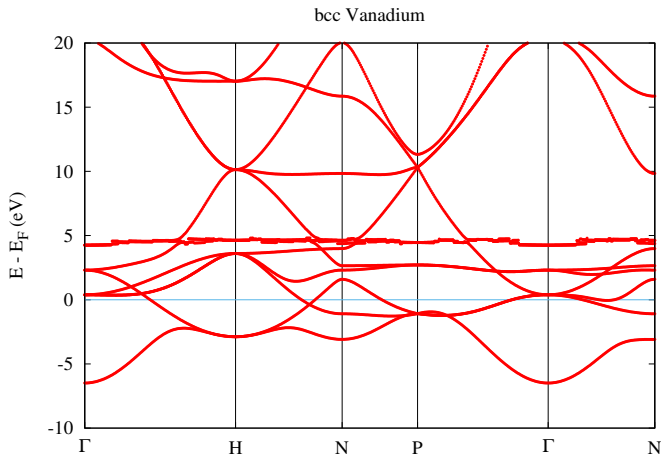
- $R_{MT} = 2.20 a_0$ , lostElectrons = 0.100

# Semicore states and ghost bands



- $R_{MT} = 2.17 a_0$ , lostElectrons = 0.109

# Semicore states and ghost bands



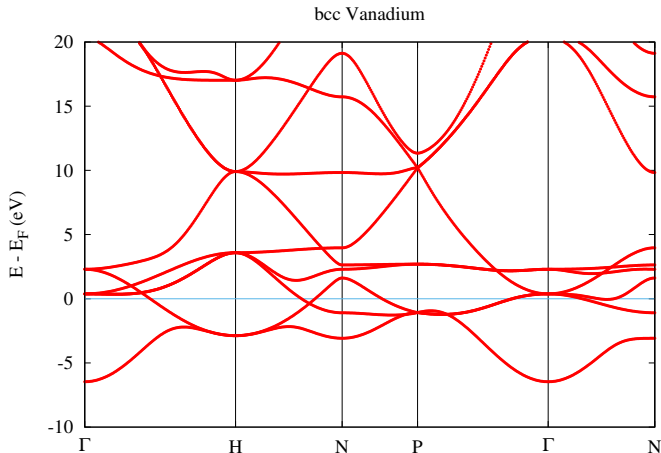
- $R_{MT} = 2.16 a_0$ , lostElectrons = 0.112

# Semicore states and ghost bands

```
*****juDFT-Error*****  
Error message:differ 2: problems with solving dirac equation  
Error occurred in subroutine:differ  
Error from PE:0/1  
*****  
Last known location:  
Last timer:Updating energy parameters  
Timerstack:  
Timer:eigen  
Timer:gen. of hamil. and diag. (total)  
Timer:Iteration  
Timer:Total Run  
*****
```

- $R_{MT} = 2.15 a_0$ , lostElectrons = 0.123
- This error message can also have other causes.
- Other error messages are also possible.

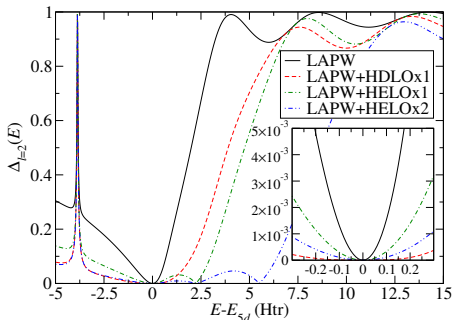
# Semicore states and ghost bands - with SCLO



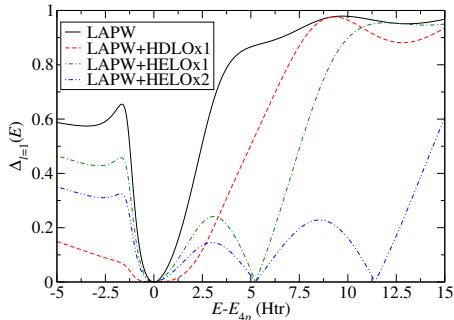
- $R_{MT} = 2.16 a_0$ , lostElectrons = 0.012, SCLO for 3p state

# Linearization error depending on energy mismatch

fcc Cerium



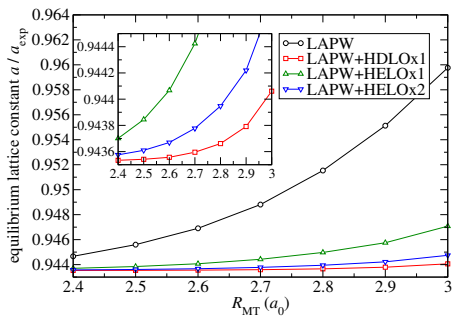
bcc Vanadium



$$\blacksquare \Delta_l = \sqrt{\| u_l(r, \epsilon) - \tilde{u}_l(r, \epsilon) \|^2}$$

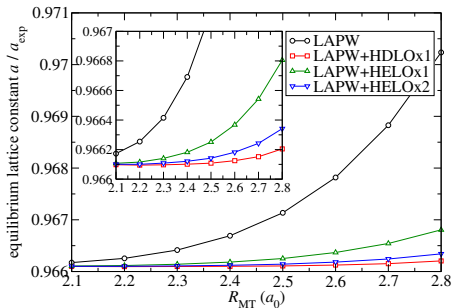
# The linearization error and MT radii

fcc Ce



- lattice constant changes by 1.6% when MT radius is reduced

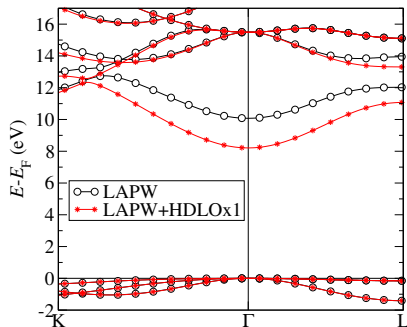
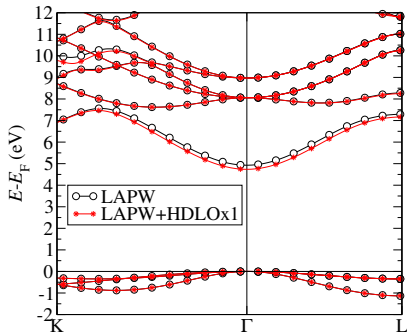
rock-salt KCl



- lattice constant changes by 0.4% when MT radii are reduced



# The linearization error and unoccupied states



- KS band gap for rock-salt KCl is reduced by 4% by adding one set of HDLOs

- KS band gap for fcc Ar is reduced by 19% by adding one set of HDLOs

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- **FLAPW in practice**

- Setting the parameters
- Semicore states and ghost bands
- The linearization error

- **Using fleur**

- The input file generator
- The inp.xml file

- **Further reading**

- **Conclusion**

# The input file generator

- Fleur uses complex input
  - Input file with default parameters is generated by input file generator **inpgen**
  - Simple structural input needed for inpgen
- Example input for inpgen

```
NaCl bulk

&lattice latsys='fcc' a0=10.62026 /

2
11 0.0 0.0 0.0
17 0.5 0.5 0.5
```

- Usage: `inpgen < myInputFile.txt`

# Input file generator - command line options

- **-h** write out list of command line options
- Optional Input to be generated
  - **-explicit** add some optional input directly to **inp.xml** file.
    - List of k points
    - Symmetry operations (otherwise if needed in **sym.out** file)
    - Noncollinear magnetism input
  - **-noco** generate noncollinear magnetism input in inp.xml
  - **-genEnpara** generate **enpara** file for more energy parameter options
  - ...

# Fleur input: the inp.xml file

```
<?xml version="1.0" encoding="UTF-8" standalone="no"?>
<fleurInput fleurInputVersion="0.30">
  <comment> Fleur is cool </comment>
  <calculationSetup> ... </calculationSetup>
  <cell> ... </cell>
  <xcFunctional> ... </xcFunctional>
  <atomSpecies> ... </atomSpecies>
  <atomGroups> ... </atomGroups>
  <output> ... </output>
</fleurInput>
```

- Usage: fleur
- **-h** command line option to display all fleur modes

# The inp.xml file - calculationSetup

```
<calculationSetup>
  <cutoffs Kmax="3.6" Gmax="10.7" GmaxXC="8.9"
    numbands="0"/>
  <scfLoop itmax="15" minDistance=".0"
    imix="Anderson" alpha=".05"/>
  <coreElectrons ctail="T" frcor="F" kcrel="0"/>
  <magnetism jspins="1" l_noco="F"/>
  <bzIntegration valenceElectrons="16.0" mode="hist"
    fermiSmearingEnergy=".001">
    <kPointCount count="15" gamma="F"/>
  </bzIntegration>
</calculationSetup>
```

+ Optional input, e.g., nocoParams, geometryOptimization

# The inp.xml file - cell, xcFunctional

```
<cell>
  <symmetryFile filename="sym.out"/>
  <bulkLattice scale="1.000" latnam="any">
    <bravaisMatrix>
      <row-1> 0.00000 5.31013 5.31013 </row-1>
      <row-2> 5.31013 0.00000 5.31013 </row-2>
      <row-3> 5.31013 5.31013 0.00000 </row-3>
    </bravaisMatrix>
  </bulkLattice>
</cell>
<xcFunctional name="pbe"
  relativisticCorrections="F"/>
```

# The inp.xml file - atomSpecies

```
<atomSpecies>
  <species name="Na-1" element="Na" atomicNumber="11"
    coreStates="1" magMom=".00" flipSpin="T">
    <mtSphere radius="2.80" gridPoints="925"
      logIncrement=".0120"/>
    <atomicCutoffs lmax="10" lnonsphr="8"/>
    <energyParameters s="3" p="3" d="3" f="4"/>
    <lo type="SCL0" l="0" n="2" eDeriv="0"/>
    <lo type="SCL0" l="1" n="2" eDeriv="0"/>
  </species>
  <species name="Cl-2" element="Cl" atomicNumber="17"
    coreStates="4" magMom=".00" flipSpin="T">
    ...
  </species>
</atomSpecies>
```



# The inp.xml file - atomGroups

```
<atomGroups>
  <atomGroup species="Na-1">
    <relPos> .000000 .000000 .000000</relPos>
    <force calculate="T" relaxXYZ="TTT"/>
  </atomGroup>
  <atomGroup species="Cl-2">
    <relPos> 1.0/2.0 1.0/2.0 1.0/2.0</relPos>
    <force calculate="T" relaxXYZ="TTT"/>
  </atomGroup>
</atomGroups>
```

+ Optional input, e.g., nocoParams

# The inp.xml file - output

```
<output dos="F" band="F" vacdos="F" slice="F">  
  <densityOfStates ndir="0" minEnergy="-0.50"  
    maxEnergy="0.50" sigma="0.015"/>  
</output>
```

- + Optional input, e.g., vacuumDOS, plotting, chargeDensitySlicing
- Full documentation of inp.xml file on [www.flapw.de](http://www.flapw.de)

# Overview on files

## with HDF5

- inp.xml
- (sym.out)
- enpara (optional)
- cdn.hdf
- mixing\_history.\*
- input for special calculations
- out, inf, out.xml

## without HDF5

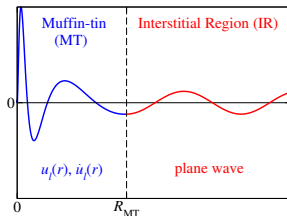
- inp.xml
- (sym.out)
- enpara (optional)
- cdn1, cdn??, cdnc
- mixing\_history.\*
- stars, wkf2
- input for special calculations
- out, inf, out.xml

# Further reading

- The FLAPW method
  - Overview book - Singh *et al.*, *Planewaves, Pseudopotentials, and the LAPW Method*, Springer
  - Initial publication - Andersen, PRB **12**, 3060 (1975)
  - FLAPW for films - Krakauer *et al.*, PRB **19**, 1706 (1979)
  - Potential calculation - Weinert, J.Math.Phys. **22**, 2433 (1981)
  - Predecessor (APW) - Slater, Phys.Rev. **51**, 846 (1937)
- Local orbitals
  - SCLOs - Singh, PRB **43**, 6388 (1991)
  - HELOs - Betzinger *et al.*, PRB **83**, 045105 (2011)
  - HDLOs - Friedrich *et al.*, PRB **74**, 045104 (2006)
  - Linearization error - Michalícek *et al.*, CPC **184**, 2670 (2013)

# Conclusions

- LAPW basis + local orbitals
- Guidelines for setting parameters
  - $K_{\max}$ ,  $I_{\max}^{\alpha}$ ,  $I_{\text{nonsphr}}^{\alpha}$
  - $R_{\text{MT}}^{\alpha}$ ,  $E_l^{\alpha}$
  - $G_{\max}$ ,  $G_{\max\text{XC}}$
- Semicore states and ghost bands
- The linearization error
- Fleur input files
- Not discussed
  - General numerical DFT parameters, e.g., k point set, Fermi smearing, ...



$$I_{\max}^{\alpha} \approx K_{\max} \cdot R_{\text{MT}}^{\alpha}$$

